ISSN 2278-8808
An International Peer Reviewed
SCHOLARLY RESEARCH JOURNAL
FOR INTERDISCIPLINARY STUDIES

ROLE OF MATHEMATICS IN LEARNING PHYSICS

Sarveshwar Kasarla

Assistant Professor of Physics, Institute of Science, Nagpur

Abstract

This paper describes the importance of mathematics while learning physics and mathematics-based difficulties in learning Physics. The students during graduation courses offering Physics and mathematics face difficulties as they lack basic mathematical skills.

Keywords: *Mathematics, Physics, Teaching, Learning, Skills, concepts learning, modelling, competency*

Introduction

Mathematics is skill based subject and these skills are essential for teaching and learning the concepts of Physics. Students offering Physics are taking Mathematics as a compulsory subject. University well knows the importance of mathematics for the study Physics. Many students at school level don't pay attention to the mathematics as it requires lot of practise to learn the problem solving skills. The time requirement for learning these skills is more as compared to the other subject. Second reason for not giving much attention to the subject is it requires the previous knowledge and skills .e.g. the student requires the knowledge of addition, subtraction, multiplication and division for finding the square root of a number. To learn the differential equation he must know the differentiation and integration. In contrary to this for the other subject like History, he don't have to even remember the ancient History for the study of Modern history

Physics is the study of matter and energy. It involves the definitions, concepts and its applications in derivations, Numerical problems .Without taking help from mathematics one can remember the definitions and descriptions but for the maximum portion of Physics like say derivation and problems it essentially requires mathematical base and logical thinking. e.g. to learn gauss's theorem in electrostatics he has to deal with differentiation ,integration, area, volume, flux concepts.

Even for the Physics teachers too mastering the mathematical skills is important. It helps in the curriculum development. When the tuning between the students and teacher matches in the classroom everybody will enjoy the subject. Using the mathematics the Physics lecture can be compressed which further saves the time of both teacher as well as students. e.g. if Maxwell's equations in electromagnetics is expressed in mathematical form only it becomes easy to remember the formulae and thus physical explanation of such equations further becomes easy to recall.

Importance of Mathematics

Learning Mathematics not only encourages logical reasoning and mental accuracy but it provides an effective way of building mental regulation and. Also, mathematical knowledge plays a important role in understanding the contents of other subjects such as science, social science, and even music and art.

Mathematics is important as far as physics is concerned. Mathematics constitutes a large portion of its language in Physics. It is necessary to note this importance and make a deliberate effort to sharpen mathematical knowledge of physics students'. When the mathematical knowledge appropriate to the teaching of the intended physics concept is required.

Mathematics is the only subject that always demands previous knowledge plus mathematical skills. During the graduation course, students are required to be skill-up with algebra, equations, differential equations integration and differentiation. Students who choose to ignore Mathematics, or not take it seriously, forgo many future career opportunities that they could have. They essentially turn their backs on more than half the job market. The vast majority of university degrees require Mathematics. The importance of Mathematics cannot be disregarded for potential future careers.

Good knowledge of Mathematics and Statistics is required in the Physical Sciences , Life and Health or Medical Sciences, Pharmacy, Social Sciences (including Anthropology, Communications, Economics, Linguistics, Education, Geography), Technical Sciences (like Computer Science, Networking, Software development), Business and Commerce, Actuarial science (used by insurance companies)and Medicine.

Career in Mathematics

Every area of Mathematics has its unique applications to the different career options. For example, Algebra is very important for computer science, cryptology, networking, the study of symmetry in Chemistry and Physics. Calculus (including differential equations) is used in Chemistry, Biology, Physics, Engineering, the motion of water (hydrodynamics), rocket science, molecular structure, option price modelling in Business and Economics models, etc.

As the students lack a depth of mathematical competency, understanding of physics tends to be obscured by the students' attempt to understand the mathematics, which is used to develop the logical arguments that bring about an understanding of the intended physics concepts, which Ausubel (1963, 1968) describes as meaningful learning. The lack of mathematical competency could lead to an over-emphasis on qualitative methods by physics teachers. But, such an approach would necessarily be limited in scope, since certain aspects of physics explanations are rooted in and explicated through the mathematics knowledge domain (Nashon, 2006).

Physics content

Students opting for physics have to face two types of material while studying.

1. Memory based: Definitions, descriptions e.g. Statement of Newton's laws, Description of Cathode ray oscilloscope. Even logical thinking helps in the description with the diagram.

2. Logical and Mathematical skills-based: Derivations and Numerical problems.

e.g. proof of Gauss's theorem, derivation of expression for the Electric Intensity

The weightage given in the first part is very less as compared to the second part.

Thus the student has to prepare a logical-mathematical base by heart.

Connection between Mathematics and Physics

According to Weizsacker and Juilfs (1957), "The tool of conceptual thought in physics is mathematics, for physics treats the relations measured, which is numerically determined, magnitudes" (p. 11). The importance of mathematics in teaching physics is evident and is recognized so by the teachers of physics. It is very clear from the curriculum of Physics. There is hardly a single page of Physics without a single equation or other forms of mathematical expression. One cannot acquire complete physics knowledge without its quantitative aspects (Nashon, 2006). Physics knowledge domain is constructed through both qualitative (description, observation) and quantitative (measurements, calculations) methods.

Many problem-solving tasks in physics are characterized by the use of equations and other forms of formulae. In our view, students coming to physics classes where instructions utilize knowledge of equations they already know experience minimum obscurity of the intended physics concepts by the mathematics. Conversely, if too much new information is to be learned concurrently or over a too short time, students may experience cognitive overload.

The use of formal expressions in physics is not first, a matter of rigorous and routinized applications of principles, followed by the formal manipulations to obtain an answer. Rather, successful students learn to understand what equations say in a fundamental sense; they have a feel for expressions, and this guides their work

Role of mathematics

The language of physics is mathematics. To study physics seriously, one needs to learn mathematics that took generations of brilliant people centuries to work out. Algebra, for example, was cutting-edge mathematics when it was being developed in Baghdad in the 9th century. But today it's just the first step along the journey.

Algebra

Algebra provides the first exposure to the use of variables and constants, and experience manipulating and solving linear equations of the form y = mx + b and quadratic equations of the form $y = ax^2+bx+c$. Such equations are used kinematical equations to find the position, time, velocity, and acceleration of the particle.

Geometry

Geometry at this level is two-dimensional Euclidean geometry, Courses focus on learning to reason geometrically, to use concepts like symmetry, similarity, and congruence, to understand the properties of geometric shapes in a flat, two-dimensional space. In Crystallography this concept is useful.

Analytic Geometry

Analytic geometry is the marriage of algebra with geometry. Geometric objects such as conic sections, planes, and spheres are studied by the means of algebraic equations. Vectors in Cartesian, polar and spherical coordinates are introduced.

Trigonometry

Trigonometry begins with the study of right triangles and the Pythagorean theorem. The trigonometric functions sin, cos, tan, and their inverses are introduced and clever identities between them are explored.

Calculus

Calculus begins with the definition of abstract functions of a single variable and introduces the ordinary derivative of that function as the tangent to that curve at a given point along the curve. Integration is derived from looking at the area under a curve, which is then shown to be the inverse of differentiation.

Multivariable calculus introduces functions of several variables f(x,y,z...), and students learn to take partial and total derivatives. The ideas of directional derivative, integration along a path, and integration over a surface are developed in two and three-dimensional Euclidean space.

Ordinary Differential Equations

This is where physics begins! Much of physics is about deriving and solving differential equations. The most important differential equation to learn, and the one most studied in undergraduate physics, is the harmonic oscillator equation, $ax'' + bx' + cx = f(t)$, where x' means the time derivative of $x(t)$.

Partial Differential Equations

For doing physics in more than one dimension, it becomes necessary to use partial derivatives and hence partial differential equations. The first partial differential equations students learn are the linear, separable ones that were derived and solved in the 18th and 19th centuries by people like Laplace, Green, Fourier, Legendre, and Bessel.

Linear Algebra

In linear algebra, students learn to solve systems of linear equations of the form $ai_1x^1 + ai_2\, x^2 + \ldots + ai_n\, x^n = ci$ and express them in terms of matrices and vectors. The properties of abstract matrices, such as inverse, determinant, characteristic equation, and of certain types of matrices, such as symmetric, antisymmetric, unitary, or Hermitian, are explored.

Methods of approximation

Most of the problems in physics can't be solved exactly in closed form. Therefore we have to learn techniques for making clever approximations, such as power series expansions, saddle point integration, and small (or large) perturbations.

Probability and statistics

Probability became of major importance in physics when quantum mechanics entered the scene. A course on probability begins by studying coin flips, and the counting of distinguishable vs. indistinguishable objects. The concepts of mean and variance are developed and applied in the cases of Poisson and Gaussian statistics.

Here are some of the topics in mathematics that a person who wants to learn advanced topics in theoretical physics, especially string theory, should become familiar with.

Real analysis

In real analysis, students learn abstract properties of real functions as mappings, isomorphism, fixed points, and basic topologies such as sets, neighborhoods, invariants, and homeomorphisms.

Complex analysis

Complex analysis is an important foundation for learning string theory. Functions of a complex variable, complex manifolds, holomorphic functions, harmonic forms, Kähler manifolds, Riemann surfaces, and Teichmuller spaces are topics one needs to become familiar with the string theory.

Group theory

Modern particle physics could not have progressed without an understanding of symmetries and group transformations. Group theory usually begins with the group of permutations on N objects and other finite groups. Concepts such as representations, irreducibility, classes, and characters.

Differential geometry

Einstein's General Theory of Relativity turned non-Euclidean geometry from a controversial advance in mathematics into a component of graduate physics education. Differential geometry begins with the study of differentiable manifolds, coordinate systems, vectors, and tensors. Students should learn about metrics and covariant derivatives, and how to calculate curvature in coordinate and non-coordinate bases.

Differential forms

The mathematics of differential forms, developed by Elie Cartan at the beginning of the 20th century, has been powerful technology for understanding Hamiltonian dynamics, relativity, and gauge field theory. Students begin with antisymmetric tensors, then develop the concepts of exterior product, exterior derivative, orientability, volume elements, and integrability conditions.

Students Difficulties

Perhaps the hardest class for most home school parents and professional teachers to teach properly is Mathematics. This is because few among us have truly learned Maths; most of us struggled through the courses. One cannot let most students simply follow a textbook, since this only provides half of the information needed to properly solve Math problems.

Math is very different from English or History or even Science because Math is the only subject that shares two characteristics:

1. You must start at the beginning and build upon what you learned in the previous lesson.
2. The answer is 100% right or 100% wrong, there is no such thing as a partially correct answer.

In other classes such as History, one can start at any point and study just that particular part of History. In English, there are various degrees of correctness and opinions may differ as to what is correct. But not in Math: Math is precise and requires exact execution. Math answers are either right or they are wrong. ("Partial credit" is merely a device used by teachers from the 1970s who were too concerned with the short-term self-esteem of their pupils.)

Difficulties in Mathematics

1. Answer in mathematics is either right or wrong.

Most students who get into serious Mathematics have been taught that the answers that they are expected to come up with are either correct or wrong. The partial correct answer has no weightage in the part of the examination. The student may improve the errors in the mathematical procedure to find the correct answer. The ultimate goal must be to get the correct answer only. The discussions thus end with the proof and not with the opinion.

2. Mathematics is composed of basic Building Blocks.

Unlike other areas of study, one can't just pick up a Math book at the college level and begin unless the proper sequence of preliminary courses has been taken. This is because the study of Mathematics is the study of a procedure and a methodology not a set of facts or opinions. A body of facts, such as History or Geography, can be entered at many points. A methodology must be taught from the beginning.

In Mathematics, simple routines and terminology must be learned before advanced routines and terminology can be learned. Any student that is not capable of 99% accuracy on basic addition, subtraction, and multiplication is going to have tremendous problems with long division simply because multiplication and subtraction are the basis of division. Similarly, any student that has trouble with fractions will have trouble with algebra -- fractions are a vital part of algebraic manipulation.

3. Poor Process Causes Most Problems

Student Math difficulties fall into three categories -- Conceptual, Algorithmic, and Process. If the student has a conceptual problem with negative numbers. and their manipulation. He didn't clearly understand the idea of zero. Other concepts that are commonly missed include basic geometrical concepts such as area, volume, diameter, etc; the fraction as division; exponents; logs; trigonometric functions; and other items that are covered quickly. Similarly, students -- particularly those who miss class -- can miss a critical concept that can cause problems for years to come. Fixing Conceptual problems is simplest -- explain the concept to the student until they understand it.

4. Neatness Counts

In Math, Neatness makes a big difference once 4-digit addition is introduced. The student must keep the columns in alignment to get the right answer consistently. A sloppy student will accidentally misalign columns of numbers and add them incorrectly.

There are several points where neatness must be upheld in Maths:

- Columns must be neatly aligned vertically -- particularly in long division.
- Decimal points must be aligned vertically in arithmetic problems.
- Numerals and variable letters must be precisely and neatly written.
- If unclear, the numeral "7" should be written with a dash across the midpoint to clearly distinguish from "1".
- In the same manner, "Z" should be written with a slash to distinguish from "2".
- When solving equations, each step should be written on a separate line.
- "=" signs should be lined up vertically.
- After the introduction of variables, the "x" sign should be replaced by the "*" for multiplication.
- After the introduction of fractions, the fraction bar should replace the simple division sign.
- Solve equations 1 step at a time. Avoid trying to do multiple steps at once because you will get confused.
- Only 1 equal sign should exist in any equation.

5. Repeating the same procedure Each Time

One of the common problems that develop both in the school years is the Process problem where the student approaches each problem as a different problem. This leads to confusion and a tendency to get the process right once, but then to drift off course. The solution is " Do It The Same Way Each Time".

Encourage your student to look at categories of problems. Each type of problem has one method to solve that problem. Emphasize that once a student finds a way to solve a category of problem, he or she can use that method to solve ALL problems that are in the category. Then, use that method over and over and over again. If the student understands and uses this Concept, then they will concentrate on the higher-level question of "which category is this problem a member of?"

DITS-WET is particularly good for students that have difficulty with multi-step problems (such as long division or basic algebra), and with word problems. Imitate a machine or a computer as you do a sample problem (such as an 8-digit addition problem). Emphasize DITS-WET each cycle of the problem.

6. Shortcuts

Many students of Math fall into a common trap.A simple problem is presented as an introduction to a class of problems. The brighter students quickly realize that the problem can be solved by mental shortcuts and don't study the method rigorously. A few weeks or months later, a much more complicated instance of the problem appears and the shortcuts don't work because the complexity is too great.

Students need to learn the methods that always work. Teaching shortcuts is something that should only be done with students who have demonstrated a mastery of the complete method. a simple method is used to solve a simple problem. But the method doesn't work with more complex problems. In effect, the student must learn several methods to solve problems of increasing complexity, when one moderately complex method would always solve any of the problems. In the long term, it is to the student's benefit to have one thoroughly understood method that always works, than be faced with the added complexity of having several methods for different situations..

7. Check your Answer

Many students never check their answers. It's particularly important as multi-step problems appear. And reversing the process can check almost any problem.

The best way to teach checking is to require it. Require a student to solve the problem and put the reverse check beside the problem.

8. Units are Important

Many students don't work with units for several years after they are introduced. Units tend to be added at the end of the problem as an afterthought rather than pulled through the method with the numbers. The result is that many mistakes, which could be caught by paying attention to units, are not caught.

For example, if a student divides 3 hours by 60 Kilometer to get kmph, he will catch this mistake. if he uses his units, but may not catch it if he simply adds "kmph" at the end.

Require units to be pulled through the entire problem and count the problem wrong if they are not.

9. Ambient Atmosphere

No student can develop the step-by-step concentration that Maths requires in an environment surrounded by rock music and television. That is because modern music and TV are designed to grab and hold the listener or viewer's attention.

Mathematicians almost without exception have noted that the best environment for doing difficult Maths problems is either a quiet place (Newton invented calculus sitting outside under the trees.) or referring to classical music(Santoor vaadan, sitar vaadan, etc) as a background to seatwork.

10. Maths is a Toolbox

A good workman needs tools. And he needs the right tools for the job. In any profession, better tools are invented as time progresses -- tools that allow difficult tasks to be handled quickly and easily. For example, today's modern carpenter begins work with a hammer, nails, and a handsaw(Simple tools). Yet he has the advanced tools available. it just took longer and took more effort to complete the task than it would have if he had done with advanced tools.

In the same way, mathematicians started with addition and subtraction. Multiplication was simply a tool that sped up addition. Algebra is a wonderful tool for solving many different types of problems and trigonometry saves considerable time and effort in calculating heights and elevations for surveyors. Calculus was fully developed to make solving the motion of the planets much easier but is invaluable to civil engineers and economists. And complex mathematics considerably simplifies the development of complex electronic circuits.

11. Need of Persistence and Habits

The keys to success in Math a step-by-step approach and mentality is required. The Brilliance and Intuition in Mathematics is required for new discoveries like Newton did with Calculus, Godel with Incompleteness, and Einstein with Tensor Calculus. But the task of Mathematics up until at least the level of Differential Equations is the task of the good workman.

Mathematics success is driven by Persistence. Ideally, solving a complex algebra problem should have the same feel as architect a house -- gradually the outlines of the beautiful ending appear as the step-by-step process occurs. Mathematics has a Concept of Elegance, which is defined as a step-by-step solution to a problem that is clear, easy to follow, as short as possible, and comes quickly to the correct answer. As you paint a house, you cover the walls step by step. When you solve a Mathematics problem, you go step by step.

Conclusions

Students who wish to pursue their career in Physics or Mathematics are advised to take mathematics seriously from the school level and devote the sufficient time to acquire all the skills essential for the future studies. Teachers should use mathematics in the classroom teaching to make students well aware of the language of mathematics.

Students must opt for mathematics in the university with a good background in Mathematics(knowledge and skills). It is well known that students who enters university with a poor mathematical skills have a difficult time progressing in the disciplines they have chosen . So it strongly recommend students take Mathematics seriously during their school years and score at least a 50 % to be able to do reasonably well in university Mathematics.

In the competition world where the recruitment ids done by conducting examinations (Civil Services, State services, Banking, etc.) become a serious problem for students if students don't do well in Mathematics. Thus they exclude themselves from the many career paths that need Mathematics. All our students to take matters into their own hands, to study and practise hard, achieve a level of excellence in any of the Physical, Social, Health Sciences, Business, Medicine, or related areas.

The basic mathematical procedure should be learned and practiced with 100 % accuracy. Even though it is a time consuming process it can't be avoided. The numerical problems of different varieties and units should be practiced frequently. The physics concepts behind the problems should be marked. Neatness should be maintained while solving the steps. Shortcuts should be avoided at the primary stage of learning but after a clear idea of concepts use of shortcuts admitted to getting the precise and accurate solution.

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